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THE PLACE OF ELEMENTARY CALCULUS IN SENIOR HIGH SCHOOL MATHEMATICS

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Various articles which have appeared in THE MATHEMATICS TEACHER during the past year relative to the present-day status of mathematics in our secondary schools and in the first part of college work show that the mathematics curriculum is in a state of transition. These articles point out that in secondary mathematics the aim is to make the subject contribute as much as possible "to a better appreciation of civilization and a better understanding of life about us." Another and more striking modification is the new principle stated in the Report on College Entrance. It says that the preparation in mathematics should be the kind that will be the best basis for work in all the main departments in college rather than that the preparation should show merely the mastery of certain standard parts of abstract mathematics. The Report of the National Committee on Mathematical Requirements is especially interesting in that it recommends as electives in senior high school mathematics simple courses in elementary calculus and in the history of mathematics. This is in harmony with the trend in the status of mathematics as I found it apparently to be from a year's graduate study which involved an investigation also along this line. I shall put some of the results of my investigation in the form of conclusions in the hope that they may be of interest to the reader and may aid those in doubt to decide in favor of these new departures in the mathematics curriculum.

I. *The Status of Mathematics in Foreign Schools Corresponding to our High Schools.*

The main types of the material used as a basis for this investigation were the leading European textbooks on elementary calculus which have been written for use in their secondary schools, the class lists or programs of study of certain particular schools as the Manchester Grammar School of England and the Lycées of France, and a comparison of the curricula in mathematics as found in different countries.

In at least ten of these foreign countries, calculus is taught before the end of the year which corresponds to our twelfth school year to pupils whose ages range from fifteen to eighteen years. It also is apparent that the United States is far behind in the work that is being done in mathematics in our secondary schools. One of the main reasons why the pupils in the schools abroad are able to accomplish this work in mathematics which throws them two years ahead of our pupils by the end of the twelfth school year is because of the more profitable arrangement of the curricula in mathematics in their schools. A more profitable arrangement has already been made in the mathematics curriculum of our standard junior high school and an attempt is now being made to do a similar thing for the mathematics curriculum of our senior high school.

II. *The Trend of Mathematics in our Public School System.* In colonial times, in the schools corresponding approximately to our high schools, the arithmetic consisted of operations with integral numbers and of some work in fractions and proportion. A few decades later, when our high schools first began to make their appearance, arithmetic had become more difficult and included such topics as stocks and exchange, circulating decimals, alligation, cube root, and permutations and combinations. Then it was realized that it would be better not to teach so much of the more difficult parts of arithmetic but more of the simpler parts of some of the higher branches in mathematics. Hence, during the latter part of the nineteenth century, algebra and geometry became a part of the high school mathematics, and these were followed a few years later by the introduction of trigonometry.

We are again passing through a transition period in the mathematical content of our curricula analogous to the one just described. According to the arrangement of the work in mathematics in the standard junior high school, propaedeutic courses are given in algebra, geometry, and trigonometry by the end of the ninth school year. The nature of the subject-matter in these three courses differs from the traditional textbook type. Instead of an unnecessary amount of time and space being given to puzzle problems, problems of mere complexity, and problems of a too advanced character merely to make the work exhaustive, the time and space are devoted to a simple, careful, and helpful introduction into these subjects.

This preparation in mathematics in the junior high school makes it possible to complete all the desirable present high school mathematics by the end of the eleventh school year. Since this arrangement leaves room for an elective course in elementary calculus in the twelfth school year, the question arises, is this subject of sufficient importance?

III. *The Important Position Occupied by Calculus in the Mathematics Structure.*

Those who are familiar with the courses in engineering know that calculus furnishes so many of the fundamental principles in the theory of engineering that much of the work in this field would be impossible without calculus. A similar situation exists in the fields of physics and mechanics. Calculus also plays an important part in the mathematics of electricity, light, shipbuilding, aviation, wireless telegraphy, the chemistry of heat, and the theory of probability in statistics. In the field of pure mathematics, calculus is used extensively in differential equations, integral equations, the theory of the functions of a complex variable, and the theory of relativity. In brief, calculus is one of the most important aids in applied mathematics and contributes largely to the field of pure mathematics. Thus calculus forms *the* connecting link between the fields of applied and pure mathematics *and* the school work in the reorganized mathematics preceding the last year in the senior high school. Therefore, because of the important position occupied by calculus in the mathematics structure, twelfth year pupils who are interested in mathematics should be given the opportunity to become acquainted with the subject provided it is not too difficult.

IV. *A Historical Survey of the Natural Growth of Calculus in the Development of Mathematics.*

In this survey the chronological order was followed. The first general step in the development of calculus is the method of exhaustion and was used as early as 400 B. C. The remaining general steps originated in the seventeenth century, and the first textbook on differential calculus was published during this century (1696). After Cauchy had given the first rigorous deduction of the derivative as a limit (c. 1820), calculus began to be rapidly extended in an increasing number of ways in both applied and pure mathematics. This study of the primitive methods in calculus shows that a first course in elementary calculus lies well within the mental grasp of twelfth year pupils in mathematics.

V. *A Comparison of the Leading Textbooks on Elementary Calculus for Beginners and for Self-Instruction.*

The English authors lead in this type of book. They keep in mind the needs of the learner and they keep the theory closely allied with work in physics and mechanics. There are also a number of German textbooks of this type, but the German authors adhere more closely to the scientific method and include too much work of a purely abstract nature. The French combine their work in elementary calculus with that of their advanced courses in algebra. There are almost no American textbooks on elementary calculus which have been written for others than college students. This comparison of textbooks shows that a number of the best teachers of secondary mathematics do not consider elementary calculus too difficult for their pupils.

VI. *The Trend of American Education in General.*

The trend of American education in general shows that the length of the school term has increased from that of a few months in the early days of the public school system to a term of nine or ten months with compulsory education under certain conditions. The number of years that the average child attends school has likewise increased. It is also true that many pupils besides those who go to college do not cease their educational activity at the close of their public school life. This is indicated by the great number of people taking correspondence courses and extension courses, and by the great number of evening schools and shop schools. The success of chautauquas also shows that the public is still athirst for knowledge. Indeed, education can no longer be thought of so much as a preparation for life but rather as a part of life itself for many years. However, after the public school period, the person who studies along any particular line no longer has the benefit of the teacher's guidance. Hence, those who wish to study further along the line of mathematics will find desirable a preparatory knowledge of differentiation and integration such as would be given in a course in elementary calculus in the senior high school.

VII. *General Conclusions.*

The study of this problem also shows that the proposed course in elementary calculus is useful; that it generates an appreciation of mathematics; that it develops the kind of thinking that can be applied to other situations in life; that it is interesting to the pupils who elect mathematics; and that it is not too difficult for such pupils.

The theory and applications of elementary calculus can be so simplified that it becomes an interesting and extremely valuable subject for such pupils in the senior high school who show fair ability in mathematics.

It is evident that a course in elementary calculus should be included in the *elective* mathematics of the senior high school for the following reasons:

It is necessary if our pupils are to have an equal opportunity in mathematics with pupils in schools abroad;

Such a course is needed to satisfy the demand for additional mathematics in the senior high school which has been created by the trend of mathematics in our public school system;

Calculus is one of the most important aids in applied mathematics and contributes largely to the field of pure mathematics;

Such a course will enable the pupil who intends to enter any field of applied mathematics to continue further his study of calculus as found in such a field without the aid of a teacher.